

LOGARITHMIC PRESENTATION OF MUSICAL INTERVAL IN BACH'S TUNING

Karel Irmann

Slunecna 824, Horovice CZ 26801

irmannkarel@seznam.cz

Abstract: Musical interval is a distance between two tones which can be defined by the difference of decimal logarithms of frequency this two tones. Presented project explicitly numerically defines how Bach's tuning chase size of the interval of the major third.

Bach's tuning C – komplex	J.S. Bach (1685 – 1750)	
Ges4 - Bb4 = 0,10235	Des4 – F4 = 0,10235	As4 – C5 = 0,10235
Es4 - G4 = 0,10184	Bb4 - D5 = 0,10034	F4 - A4 = 0,09884
C4 - E4 = 0,09708	G4 - B4 = 0,09683	D4 - Fis4 = 0,09835
A4 - Cis4 = 0,09984	E4 - Gis4 = 0,10159	B3 - Dis4 = 0,10234
5/4 = 0,09691	81/64 = 0,10230	

1 Elaboration

The Bach's irregularly tempered tuning was documented by the composition cycle „Das wohltemperierte Klavier“.

In the Table 1., the division of the musical extent in octaves is shown. The tone frequencies are in Hz, corresponding are indicated to the frequencies. The size of the octave interval of 0,30103 is defined by the difference of the decadic logarithms of the frequencies indicated.

In the Table 2., one octave A4 - A5 (a1 – a2) is dividend in twelve halftone steps corresponding to the Bach's tuning. The deviations from the regularly tempered tuning in cents were obtained from the Dr. Kelletat's data for the company „Wandel und Goltermann“.

In the Table 3., the major thirds are shown in the semence of the quint-fourth circle. The Cdur major thirds are indicated appr. In the centre of the table. The values of the difference of the logarithms indicated approach to the value of the natural major third $5/4 = 0,09691$. At the beginning and end of the table, the values approach to the major third $81/64 = 0,10230$.

This finding corresponds to the acoustic evaluation of the Bach's tuning mentioned in specialized musical literature.

Table 1. The division of the musical extent in octaves

TONE		FREQUENCY	LOGARITHM	TABLE 1
MUSIC	TECHN.			OCTAVE LOGARITHM
1A	A1	55,00	1,7403627	
A	A2	110,00	2,0413927	0,30103
a	A3	220,00	2,3424227	0,30103
a ¹	A4	440,00	2,6434527	0,30103
a ²	A5	880,00	2,9444827	0,30103
a ³	A6	1760,00	3,2455127	0,30103
a ⁴	A7	3520,00	3,5465427	0,30103

Table 2. Historical temperaments Bach – tuning

Tuning deviation opposite equal temperament

Octave A4 - A5 (a¹ - a²)

$$2,9444827 - 2,6434527 = 0,3010300$$

Music	NOTES		Frequency tempered	Deviation in cent	Frequency by Bach	Logarithm by Bach
	Tech.1	Tech.2				
a ¹	A4	A4	440,00	0	440,00	2,6434527
b ¹	Ais4	B ^b 4	466,16	+4	467,24	2,6695400
h ¹	B4	B4	493,88	-6	492,17	2,6921151
c ²	C5	C5	523,25	+8	525,67	2,7207132
cis ²	Cis5	Cis5	554,37	-2	553,73	2,7432981
d ²	D5	D5	587,33	+4	588,69	2,7698867
es ²	Dis5	Es5	622,25	+2	622,97	2,7944671
e ²	E5	E5	659,26	-5	657,35	2,8177967
f ²	F5	F5	698,46	+6	700,88	2,8456437
fis ²	Fis5	Ges5	739,99	-4	738,28	2,8682211
g ²	G5	G5	783,99	+8	787,62	2,8963167
gis ²	Gis5	As5	830,61	0	830,61	2,9193972
a ²	A5	A5	880,00	0	880,00	2,9444827

Table 3. The major third by Bach

THE MAJOR THIRD BY BACH					TABLE 3	
NOTES		Frequency tempered	Deviation in cent	Frequency by Bach	Logarithm by Bach	Δ logarithm by Bach
TECHN.	MUSIC					
Ais4	B ^b 4	466,16	+4	467,24		0,10235
Fis4	Ges4	369,99	-4	369,14	↑	
F4	F4	349,23	+6	350,44		0,10235
Cis4	Des4	277,18	-2	276,86	↑	
C5	C5	523,25	+8	525,67		0,10235
Gis4	As4	415,30	0	415,30	↑	
G4	G4	392,00	+8	393,81		0,10184
Dis4	Es4	311,13	+2	311,49	↑	
D5	D5	587,33	+4	588,69		0,10034
Ais4	B ^b 4	466,16	+4	467,24	↑	
A4	A4	440,00	0	440,00		0,09884
F4	F4	349,23	+6	350,44	↑	
E4	E4	329,63	-5	328,68		0,09708
C4	C4	261,63	+8	262,84	↑	
B4	B4	493,88	-6	492,17		0,09683
G4	G4	392,00	+8	393,81	↑	
Fis4	Fis4	369,99	-4	369,14		0,09835
D4	D4	293,66	+4	294,34	↑	
Cis5	Cis5	554,37	-2	553,73		0,09984
A4	A4	440,00	0	440,00	↑	
Gis4	Gis4	415,30	0	415,30		0,10159
E4	E4	329,63	-5	328,68	↑	
Dis4	Dis4	311,13	+2	311,49		0,10235
B3	B3	246,94	-6	246,09	↑	

$5/4 = 0,09691$

$81/64 = 0,10230$

Literature

- [1] Dr. Kellertat: Ein Beitrag zur musikalischen Temperatur der Musikinstrumente vom Mittelalter bis zur Gegenwart. März 1966, Emingen. Herausgegeben von Wandel und Goltermann, Reutlingen.