LOGARITHMIC PRESENTATION OF MUSICAL INTERVAL IN BACH´S TUNING

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Abstract: Musical interval is a distance between two tones which can be defined by the diference of decimal logarithms of frequence this two tones. Presented project explicitly numericaly defines how Bach´s tuning chase size of the interval of the major third.

Bach´s tuning C – komplex	J.S. Bach (1685 – 1750)	
Ges4 - Bb4 = 0,10235	Des4 – F4 = 0,10235	As4 – C5 = 0,10235
Es4 - G4 = 0,10184	Bb4 - D5 = 0,10034	F4 - A4 = 0,09884
C4 - E4 = 0,09708	G4 - B4 = 0,09683	D4 - Fis4 = 0,09835
A4 - Cis4 = 0,09984	E4 - Gis4 = 0,10159	B3 - Dis4 = 0,10234
5/4 = 0,09691	81/64 = 0,10230	

1 Elaboration

The Bach´s irregularly tempered tuning was documented by the composition cycle „Das wohltemperierte Klavier".

In the Table 1., the division of the musical extent in octaves is shown. The tone frequencies are in Hz, corresponding are indicated to the frequencies. The size of the octave interval of 0,30103 is defined by the diference of the decadic logarithms of the frequencies indicated.

In the Table 2., one octave A4 - A5 (a1 – a2) is dividend in twelwe halftone steps corresponding to the Bach´s tuning. The deviations from the regularly tempered tuning in cents were obtained from the Dr. Kelletat´s data for the company „ Wandel und Goltermann".

In the Table 3., the major thirds are shown in the semence of the quint-fourth circle. The Cdur major thirds areindicated appr. In the centre of the table. The values of the diference of the logarithms indicated aproach to the value of the natural major third 5/4 = 0,09691. At the beginning and end of the table, the values aproach to the major third 81/64 = 0,10230.

This finding corresponds to the acoustic evaluation of the Bach´s tuning mentioned in specialized musical literature.

Table 1. The division of the musical extent in octaves

TONE		FREQUENCY	LOGARITHM	TABLE 1
MUSIC	TECHN.			OCTAVE LOGARITHM
1A	A1	55,00	1,7403627	0,30103
A	A2	110,00	2,0413927	0,30103
a	A3	220,00	2,3424227	0,30103
a^1	A4	440,00	2,6434527	0,30103
a^2	A5	880,00	2,9444827	0,30103
a^3	A6	1760,00	3,2455127	0,30103
a^4	A7	3520,00	3,5465427	

Table 2. Historical temperaments Bach – tuning
Tuning deviation oposite equal temperament
Octave A4 - A5 (a^1 - a^2)
2,9444827 - 2,6434527 = 0,3010300

NOTES			Frequenncy	Deviation	Frequency	Logarithm
Music	Tech.1	Tech.2	tempered	in cent	by Bach	by Bach
a^1	A4	A4	440,00	0	440,00	2,6434527
b^1	Ais4	B^b4	466,16	+4	467,24	2,6695400
h^1	B4	B4	493,88	-6	492,17	2,6921151
c^2	C5	C5	523,25	+8	525,67	2,7207132
cis^2	Cis5	Cis5	554,37	-2	553,73	2,7432981
d^2	D5	D5	587,33	+4	588,69	2,7698867
es^2	Dis5	Es5	622,25	+2	622,97	2,7944671
e^2	E5	E5	659,26	-5	657,35	2,8177967
f^2	F5	F5	698,46	+6	700,88	2,8456437
fis^2	Fis5	Ges5	739,99	-4	738,28	2,8682211
g^2	G5	G5	783,99	+8	787,62	2,8963167
gis^2	Gis5	As5	830,61	0	830,61	2,9193972
a^2	A5	A5	880,00	0	880,00	2,9444827

Table 3. The major third by Bach

THE MAJOR THIRD BY BACH						TABLE 3	
NOTES		Frequency	Deviation	Frequency		Logarithm	Δ logarithm
TECHN.	MUSIC	tempered	in cent	by Bach		by Bach	by Bach
Ais4	B^b4	466,16	+4	467,24		2,66954	0,10235
Fis4	Ges4	369,99	-4	369,14	↑	2,56719	
F4	F4	349,23	+6	350,44		2,54461	0,10235
Cis4	Des4	277,18	-2	276,86	↑	2,44226	
C5	C5	523,25	+8	525,67		2,72071	0,10235
Gis4	As4	415,30	0	415,30	↑	2,61836	
G4	G4	392,00	+8	393,81		2,59528	0,10184
Dis4	Es4	311,13	+2	311,49	↑	2,49344	
D5	D5	587,33	+4	588,69		2,76988	0,10034
Ais4	B^b4	466,16	+4	467,24	↑	2,66954	
A4	A4	440,00	0	440,00		2,64345	0,09884
F4	F4	349,23	+6	350,44	↑	2,54461	
E4	E4	329,63	-5	328,68		2,51677	0,09708
C4	C4	261,63	+8	262,84	↑	2,41969	
B4	B4	493,88	-6	492,17		2,69211	0,09683
G4	G4	392,00	+8	393,81	↑	2,59528	
Fis4	Fis4	369,99	-4	369,14		2,56719	0,09835
D4	D4	293,66	+4	294,34	↑	2,46884	
Cis5	Cis5	554,37	-2	553,73		2,74329	0,09984
A4	A4	440,00	0	440,00	↑	2,64345	
Gis4	Gis4	415,30	0	415,30		2,61836	0,10159
E4	E4	329,63	-5	328,68	↑	2,51677	
Dis4	Dis4	311,13	+2	311,49		2,49344	0,10235
B3	B3	246,94	-6	246,09	↑	2,39109	

5/4 = 0,09691 81/64 = 0,10230

Literature

[1] Dr. Kelletat: Ein Beitrag zur musikalischen Temperatur der Musikinstrumente vom Mittelalter bis zur Gegenwart. März 1966, Emingen. Herausgegeben von Wandel und Goltermann, Reutlingen.